

Formula Sheet

PHYS/ECE 464 (Laser Physics I)- University of New Mexico (2022)

Hermite-Gaussian Beams

$$\frac{E(x, y, z)}{E_0} = H_m \left(\frac{\sqrt{2}x}{w(z)} \right) H_p \left(\frac{\sqrt{2}y}{w(z)} \right) \frac{w_0}{w(z)} \exp \left(-i \frac{kr^2}{2q(z)} \right) \times \exp(-i[kz - (1 + m + p) \tan^{-1}(z/z_0)])$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2 \left(1 + \frac{z^2}{z_0^2} \right), \quad R(z) = z \left(1 + \frac{z_0^2}{z^2} \right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0} \quad k = n \frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\epsilon_0}{2} E_0^2$

Snell's Law $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Fresnel's Reflectivities:

Intensity (Power) reflectivity: $R = |r|^2$

$$r_{||} = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_{\perp} = -\frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

Brewster angle (from 1 to 2): $\theta_B = \tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = \sin^{-1}(n_1/n_2)$




$n \rightarrow \tilde{n} = n + ik$

Lens Transformation/Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$	$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ Lens-makers' formula:
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Fabry-Perot Transmission and Reflection (for gain $G_0 > 1$, loss $A_0 < 1$, passive $G_0 = A_0 = 1$)

$T(\theta, G_0) = \frac{G_0(1 - R_1)(1 - R_2)}{(1 - G_0\sqrt{R_1R_2})^2 + 4G_0\sqrt{R_1R_2} \sin^2(\theta)}$ $R(\theta, G_0) = \frac{(\sqrt{R_1} - G_0\sqrt{R_2})^2 + 4G_0\sqrt{R_1R_2} \sin^2(\theta)}{(1 - G_0\sqrt{R_1R_2})^2 + 4G_0\sqrt{R_1R_2} \sin^2(\theta)}$ <p>Finesse $F = \frac{\pi^4 \sqrt{R_1R_2}}{1 - \sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}} = \frac{\Delta\lambda_{FSR}}{\Delta\lambda_{1/2}}$</p> <p>Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$</p> $\theta = kd = \frac{2\pi vnd}{c}$	<p><i>Passive Symmetric Fabry-Perot:</i></p> $T(\theta) = \frac{1}{1 + \left(\frac{2}{\pi} F \sin(\theta) \right)^2}, \quad F = \frac{\pi\sqrt{R}}{1-R}$ <p><i>Photon Lifetime:</i></p> $\tau_p = \frac{\tau_{RT}}{1 - R_1R_2} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$ <p><i>Resonance Condition:</i> roundtrip phase change = $q2\pi$, or $\theta = q\pi$ ($q = \text{integer}$)</p>
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ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ $AD - BC = 1$ $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	ABCD rule for Gaussian Beams $q_2 = \frac{Aq_1 + B}{Cq_1 + D}$ where	$q(z) = z + iz_0$ or $\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda_0}{\pi n w(z)}$
medium of length d and index $n_2 = n$ immersed in vacuum ($n_1 = 1$). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$ 	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$ 	Cavity Stability Condition: $-1 < \frac{A+D}{2} < 1$ q at the starting point of the cavity with a roundtrip ABCD: $\frac{1}{q} = -\frac{A-D}{2B} - i \frac{\sqrt{1 - (A+D)^2/4}}{B}$	
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$ $R > 0$ 	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1 - n_1/n_2)/R & n_1/n_2 \end{pmatrix}$	Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c / n_g}$	

Gain and Absorption: $\gamma(\nu) = \sigma_e(\nu) \left(N_2 - \frac{g_2}{g_1} N_1 \right)$, $\alpha(\nu) = \sigma_a(\nu) \left(N_1 - \frac{g_1}{g_2} N_2 \right)$;

Emission (gain) and absorption cross-sections: $\sigma_a(\nu) = \frac{g_2}{g_1} \sigma_e(\nu)$; $\sigma_e(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)$

Line shape: $\int g(\nu) d\nu = 1$; $g(\nu_0) \approx 1/\Delta\nu$

Beer's Law: $\frac{1}{I} \frac{dI}{dz} = +\gamma I$ or $\frac{1}{I} \frac{dI}{dz} = -\alpha I$

Einstein's relations: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h \nu^3}{c^3}$; $g_1 B_{12} = g_2 B_{21}$

Lorentzian line shape:

$$g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu - \nu_0)^2 + (\Delta\nu_h/2)^2}$$

Doppler broadened line shape

$$g(\nu) = \left(\frac{4 \ln 2}{\pi} \right)^{1/2} \frac{1}{\Delta\nu_D} \exp \left[(-4 \ln 2) \left(\frac{\nu - \nu_0}{\Delta\nu_D} \right)^2 \right] \text{ with } \Delta\nu_D = \left(\frac{8kT \ln 2}{Mc^2} \right)^{1/2} \nu_0$$

Physical Constants

$c \sim 3 \times 10^8 \text{ m}\cdot\text{s}^{-1}$	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$	$e = 1.6 \times 10^{-19} \text{ C}$
$k_B = 1.380 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$	$m_e = 9.1 \times 10^{-31} \text{ kg}$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$